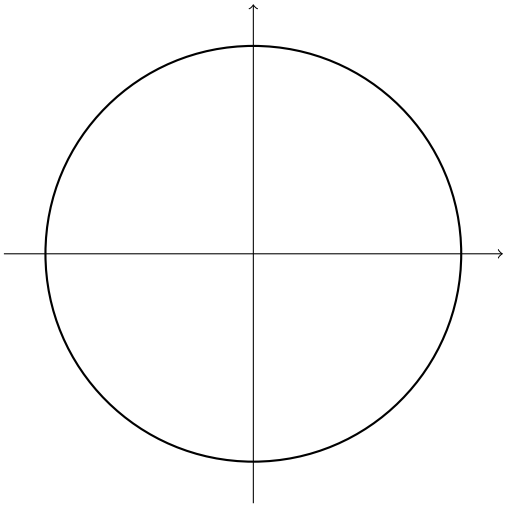
Sections 3.8 & 3.9

Implicit Differentiation

The graphs we have explored are thus far have represented function and take the form $y = f(x)$. In general, relations between x and y can be expressed *implicitly*. That is, some equation with variables x and y, like $x^2 + y^2 = 1$; the points satisfying this equation form a graph (the unit circle).



Near each point, other than $(1, 0)$ and $(-1, 0)$ this graph looks like a function, say $f(x)$. Use the chain rule to differentiate

$$x^2 + \big(f(x)\big)^2 = 1$$

$f'(x)$ is the slope of the line tangent to the graph at $(x, y) = (x, f(x))$.

Example 3.8.1. Find dy/dx for the following implicitly defined functions.

1. $xy = 1$

2. $x^3 + y^3 = 6xy$ (the graph)

3. $x^2 + xy + y^2 = 7$

4. $(3y)^{\frac{1}{3}} = x$

5. $(-2y)^{\frac{1}{-2}} = x$

6. $(ny)^{\frac{1}{n}} = x$

We seem to get every integer power of x as a derivative dy/dx this way except for $\frac{1}{x}$. Is there a function $y = f(x)$ such that $dy/dx = \frac{1}{x}$? Or, defined implicitly, $f(y) = x$ such that $dy/dx = \frac{1}{x}$? Yes there is!

Derivatives of Logarithmic and Exponential Functions

Let $y = \ln(x)$, find dy/dx.

We can also differentiate $y = b^x$ (actually without implicit differentiation!)

That helps with general logarithms: $y = \log_b(x)$

Theorem 3.9.2 (Derivatives of Exponentials and Logarithms)**.** If $b > 0$ and $b \neq 1$

$$\frac{d}{dx}e^x = e^x \qquad \frac{d}{dx}\ln x = \frac{1}{x}$$

$$\frac{d}{dx}b^x = b^x \ln b \qquad \frac{d}{dx}\log_b(x) = \frac{1}{x \ln b}$$

Example 3.9.3. Let $f(x) = x^x$ find all $a > 0$ such that $f'(a) = 0$. (Finding all horizontal tangent lines)